New Methods for the Reliability Analysis of Healthcare System Based on Application of Multi-State System

Elena Zaitseva, Vitaly Levashenko, Jozef Kostolny and Miroslav Kvassay

Abstract A healthcare system is complex and high-risk. Therefore reliability analysis of a healthcare system is principal step in its development and exploitation. The high-risk of a healthcare system is caused by different factors as human error, failure of devices and equipment, software fault, etc. These factors correlate with complex structure of a healthcare system that consists of technical and human parts. But as a rule in reliability engineering the analysis and estimation of technical components and human factor are implemented based on different methods that have different mathematical backgrounds. One of possible decision of this problem is development of new mathematical model, that allows to describe booth as technical components as human factors. Such model can be defined based on representation of a healthcare system as Multi-state System, for which can be define some (more that only two) performance levels.

1 Introduction

Reliability is a principal attribute of any system. Reliability principles are used successfully in industries to help evaluate, calculate, and improve the overall reliability of complex technical systems. As a rule complex system isn't heterogeneous and includes sub-system (components) of different types, for example, as equipment, software, human factor, organization aspects, etc. Such system can be interpreted as complex socio-technical system [20]. Reliability analysis of such system is based on

E. Zaitseva (✉) · V. Levashenko · J. Kostolny · M. Kvassay
Department of Informatics, University of Zilina, Zilina, Slovakia
e-mail: elena.zaitseva@fri.uniza.sk

V. Levashenko
e-mail: vitally.levashenko@fri.uniza.sk

J. Kostolny
e-mail: jozef.kostolny@fri.uniza.sk

M. Kvassay
e-mail: miroslav.kvassay@fri.uniza.sk

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different methods of reliability engineering. There are special methods, approaches and algorithms for quantitative analysis of reliability for every of this system components (for example, human factor, equipment, software) is realized based on special approaches of reliability theory [26]. Development of reliability analysis of socio-technical system allows extending types of analyzed system. One of new types of system is healthcare system that has principal goal to assure a patient correct treatment.

Principal specific of modern healthcare systems is intensive application of information technologies [12, 25]. The Table 1 from [25] illustrates a development of healthcare information system from an immature stage to a national stage. This is explained through entities, services and infrastructures in a defined point in time. Each stage has its own characteristics that differentiate it from other stages, but all of them have to be reliable.

The application of information technologies in healthcare cause the typical structure of a healthcare system in reliability analysis terms that has been proposed in [37]. According to this structure a healthcare system is complex and high-risk. Therefore the problem of reliability analysis and estimation of this system in design and exploitation is important. One of the first investigations of reliability in medicine was [31]. In this paper author declared principal items of reliability engineering for a healthcare system as reliability analysis of medical equipment's and devices. This tendency has been developed in most works for reliability analysis of healthcare system. For example, in papers [5, 8, 29, 30] different aspects of safety and reliability of healthcare system hardware and software are analyzed. But functioning of healthcare system (correct treatment of patient) isn't caused by reliable devices and equipment only. A human factor has high influence to healthcare system reliability too. *Human reliability analysis* (HRA) methods allow examining the probability of medical errors and risk factor for correct patient treatment [19]. Human errors for a healthcare system have been considered as independent problem [9, 19]. Therefore reliability analysis of healthcare system is separated in two independent problems: (i) reliability analysis of medical devices and equipment, and (ii) human reliability analysis of medical errors. But practical work shows that these problems are not independent and must be considered. For example, some medical error can be caused by incorrect functioning of medical devices and human medical error can influence to functioning of devices and equipment [7, 9]. In papers [37], new tendency of healthcare system reliability analysis is considered: the reliability analysis has to base on joint evaluation of all principal parts (components) of healthcare system according to these papers. This tendency supposes the application of new background for a healthcare system reliability analysis. New method and mathematical background are proposed in this paper for reliability analysis of healthcare system. The important problem of this method is development of technique that permits to investigate every system component (devices, equipment, and human factor) based on united mathematical background. This technique allows estimating a healthcare system as a whole and doesn't have to separate the investigation of human factor and technical part of system (devices, equipment, and software). The theoretical conception of this method has been presented in papers [39, 40].

Table 1 Healthcare system stage according to [25]

Stages	Entities	Department	Infrastructure
1. Hospital administration	Hospital administration	• Patient • Billing • Wards management • Diagnostics management • MIS	LAN
2. Hospital enterprise	Set of hospitals in enterprise	Stage 1+ • Finance • Materials management • HR management • Electronic claims and payments processing	Internet based access with HIPAA
3. EMR basic	Hospital + Lab + Pharmacy	Stage 2 + • Laboratory information system • Radiology information system • PACS • Pharmacy	Secure HL7 based communication
4. Clinical decision support	Stage 3 + Medical colleges	Stage 3+ • Computerized provider order entry • International codification of diseases • Alerts/ Contraindications • Used for educational purposes	Fully connected and paperless—SaaS (*Software as a service*) Model
5. Clinical research	Stage 4 + Pharma companies	• Clinical trials • Clinical data research based on drug prescriptions and reactions	OaaS2 (Operations as a service) Model + RaaS3 (Research as a service) Model
6. Regional	Primary healthcare centers + Epidemiological centers + Regional government	• Telemedicine • Aggregation of data from various hospitals at the regional level	Regional network connecting all hospitals with PHC's and Epidemiological centers
7. National	Federal government	• Data from all regions aggregated • Enables healthcare planning and government initiatives towards healthcare	National network connecting all associated service providers in the healthcare process

The development of conception proposed in [37] is continued in this paper. Important step of this conception is definition of mathematical model of healthcare system to calculate reliability indices and measures. This model is constructed based on the definition of number of system performance level and mathematical methods that are used for estimation of healthcare system reliability. The development of healthcare system mathematical model in term of reliability analysis is considered in this paper in detail in Sect. 2. This mathematical model is used for estimation of one of reliability analysis aspect that is investigation of influence of fixed component state change to a healthcare system performance level. This investigation in reliability analysis is known as importance analysis [15]. It is considered in Sect. 3. In Sect. 4 the example of simple healthcare system and its reliability analysis based on the proposed methods are presented. This system includes technical component and components that are conformed to human factors.

2 Principal Steps in Reliability Analysis of Healthcare System

Preventable medical errors are important problem in healthcare system. The persistence of medical errors according to [9, 10] suggests that there is either an absence of reliability engineering analysis or a gap in the reliability analysis currently being performed. The decision of this problem can be implemented by changing process design of healthcare system and/or performing reliability analysis of healthcare system exploitation. Typical reliability analysis of healthcare system can include the following tasks:

- Assessment of the current reliability using past data and calculation of measures for healthcare system reliability;
- Identification of weak links and allocates higher reliability goals to them;
- Analysis of weak links within the healthcare system to predict potential failures or medical errors;
- Redesign healthcare system based on critical failures identify and using process and reliability improvement techniques that will have the most impact on the outcome;
- Verification of the design improvements and calculation of measures for healthcare system reliability;
- Define reliability specifications and document in a reliability program;
- Validation of sustainability for new design of healthcare system.

According to list of these tasks, the calculation of measures and indices for healthcare system reliability is actual problem for reliability improvement of such system. Therefore the development of new methods for estimation of healthcare system reliability is actual problem too.

The estimation of reliability of any complex system as a healthcare system includes four principal steps (Fig. 1) [42]:

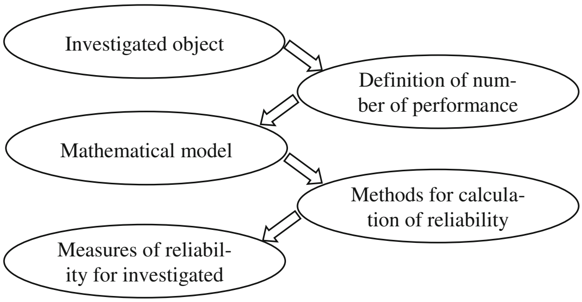


Fig. 1 The structure of a complex system reliability analysis

- the definition of number of performance levels for a system model;
- the mathematical representation of a system model;
- the development methods for the calculation of indices and measures of system reliability (for example, importance analysis);
- the measuring of the system behaviour.

Consider specifics of these steps for the healthcare system in more details.

2.1 Definition of Number of Performance Levels for Healthcare System Mathematical Model

The number of performance levels for healthcare system can be defined based on the selection one of two possible mathematical models (Fig. 2): *Binary-State System* (BSS) and *Multi-State System* (MSS).

The first of them (BSS) defines only two states for the system reliability: the functioning and failure. This mathematical model is well known and widely used in reliability engineering. The system failure can be investigated in detail based on this model. However, the analysis of other performance levels before the system failure has some difficulties based on BSS. The other mathematical model that is MSS can be used to indicate some performance levels in system reliability behaviour.

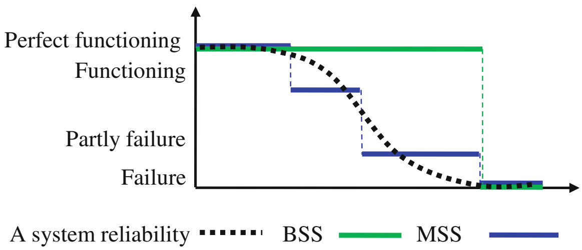


Fig. 2 Typical model in reliability analysis

elena.zaitseva@fri.uniza.sk

MSS reliability analysis is a more flexible to evaluating system reliability. MSS allows indicating more than two states for both as system and as component. It can be, for example, completely failed, partially failed, partially functioning and perfect functioning [17, 23]. However, dimension of MSS is dramatically large in comparison with BSS for equal number of system components. In addition, mathematical methods to MSS analysing is complex too. Therefore the application of MSS needs the development new special methods for the calculation of the measures and indices of system reliability.

The basic indices for MSS as system reliability, probability of system performance level, frequency have been considered in [17, 23]. In paper [15, 23, 36] some special indices, as *Importance Measures* (IMs), have been presented. These indices are quantitative estimation of the influence of one or some component states changes to system performance level. Effective methods for calculation of IMs have been proposed in paper [36] based on mathematical methods of Multiple-Valued Logic. The application of these methods for the estimation of healthcare system reliability is considered in this paper.

2.2 Healthcare System Mathematical Model

The model (mathematical representations) of healthcare system for its reliability estimation correlates with mathematical methods of reliability analysis. There are four principal groups of mathematical methods in reliability analysis (Fig. 3):

- Markov and semi-markov methods;
- Methods based on universal generation function;
- Monte-Carlo simulation;
- Structure function based methods.

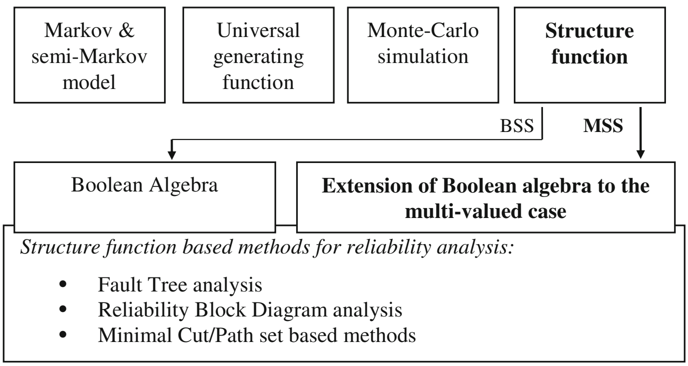


Fig. 3 Typical mathematical methods in reliability analysis

Every of these methods is used for the analysis for both as BSS as MSS and has some specifics in reliability analysis of a system. Markov model allows investigation of dynamic properties of system reliability. But Markov model dimension is increased extremely in depending of number of system components [14, 18]. The description of the system by the Universal Generating Function is used in the system reliability optimization [16]. The Monte-Carlo simulation as a rule is used for reliability assessment of system with large number of components [41]. The structure based methods were developed historically the first [3, 28]. According to these methods a system is represented and defined by the structure function. This function is defined the conformance of the system performance level and components states. As a rule for the structure function definition and representation is used Boolean functions [4, 33]. Only in some publications the structure function of MSS has been considered [13, 34]. In papers [24, 35, 36] the correlation of *Multiple-Valued Logic* (MVL) function and structure function was analyzed. The interpretation of the structure function as the MVL function allowed using the mathematical approach of MVL in the analysis of MSS structure function. The analysis the structure function and estimation of system reliability can be by such methods as fault tree analysis, minimal cut/path set analysis, etc. (Fig. 3).

The principal and important advantage of the representation of a system by the structure function is possibility to define mathematical model for a system with any structure and complexity complexity [17, 36]. Therefore the structure function of MSS is an appropriate mathematical model for healthcare system reliability analysis.

The *structure function* defines a system state (system reliability/availability/performance level) depending on the system components states. According to the definition of the structure function the system reliability in the stationary state is represented as [36]:

$$\phi(x_1, \ldots, x_n) = \phi(\boldsymbol{x}) : \{0, \ldots, m_1 - 1\} \times \ldots \times \{0, \ldots, m_n - 1\} \\ \to \{0, \ldots, M - 1\}. \tag{1}$$

In (1) the x_i is the state of the i-th system component that can be defined form 0 (the component failure) to $m_i - 1$ (the perfect component performance level): $x_i \in \{0, \ldots, m_i - 1\}$, and the system reliability has M level from 0 (as the failure) to $M - 1$ (as the perfect functioning). Note, the system component has different number of states: $m_i \neq m_k$, if $i \neq k$ $(i, k \in \{1, \ldots, n\})$. The number of the system component is declared as n.

The structure function definition (1) is the definition for MSS, where system and its component have some (more than two) performance levels. The structure function of BSS is special case of MSS structure function and can be interpreted as the Boolean function [38]:

$$\phi(x_1, \ldots, x_n) = \phi(x) : \{0, 1\}^n \to \{0, 1\}. \tag{2}$$

We will consider a coherent system in this paper. Such system has two principal assumptions for the structure function [17]: (a) the structure function (1) and (2) is monotone, and (b) the system component state decrease does not improve the system reliability.

Every system component is characterized by probability of the component state:

$$p_{i,s_i} = \Pr\{x_i = s_i\}, \quad s_i \in \{0, \ldots, m_i - 1\}. \tag{3}$$

2.3 Structure Based Method for Healthcare System Reliability Estimation

Some of methods for reliability analysis based on the structure function are shown in Fig. 1. But these methods and other structure function based methods as a rule don't allow investigating the dynamic properties of system reliability. It means that measures as system availability, reliability function and similar can be calculated. The analysis of a system performance level change has difficulties based on these methods.

In papers [35, 36] the application of Logical Differential Calculus for MSS reliability analysis has been proposed. The Logic Differential Calculus is used for analysis of dynamic properties of MVL function and this approach can be applied for analysis of dynamic behaviour of MSS that is determined by the structure function.

A system behavior and correlation of changes of components states and system reliability can be defined by mathematical tools of Logical Differential Calculus, in particular the Direct Partial Logic Derivative. The Direct Partial Logic Derivative with respect to variable x_i for the structure function (1) permits to analyse the system reliability change from j to $\bar{j}$ when the i-th component state changes from a to $\bar{a}$ [36, 38]:

$$\partial\phi(j \to \bar{j})/\partial x_i(s \to \bar{s}) = \begin{cases} 1, \textit{if } \phi(s_i, \boldsymbol{x}) = j \;\&\; \phi(\bar{s}_i, \boldsymbol{x}) = \bar{j} \\ 0, \quad \text{other} \end{cases}, \tag{4}$$

where $\phi(s_i, \boldsymbol{x}) = \phi(x_1, \ldots, x_{i-1}, s, x_{i+1}, \ldots, x_n)$; $\phi(\bar{s}_i, \boldsymbol{x}) = \phi(x_1, \ldots, x_{i-1}, \bar{s}, x_{i+1}, \ldots, x_n)$; $s_i, \bar{s}_i \in \{0, \ldots, m_i\}$, $\bar{s}_i \neq s_i$ and $j, \bar{j} \in \{0, \ldots, M\}$, $\bar{j} \neq j$.

For example, consider MSS performance level change caused by the reduction of the i-th component state. It is represented as a change of the structure function value $\phi(\boldsymbol{x})$ from state j into h. This change can be caused by the i-th component state change from s to $s-1$. In term of structure function this change is interpreted as change of the i-th variable value change from s to $s-1$. Therefore the Direct Partial Logic Derivative for MSS analysis is defined by the equation

$$\partial\phi(j \to h)/\partial x_i(s \to s-1) = \begin{cases} 1, \text{if } \phi(s_i, \boldsymbol{x}) = 1 \text{ and } \phi((s-1)_i, \boldsymbol{x}) = 0 \\ 0, \quad \text{other} \end{cases}. \tag{5}$$

Table 2 Importance measures

Short name	Description
SI	SI concentrates on the topological structure of the system and determines the probability of a system performance level change depending on the change of the *i*-th component state
BI	BI of a given component is defined as the probability that such a component is critical to MSS functioning and represents loss in MSS when the *i*-th component state reduced
CI	CI is similar to BI and take into account the probability of the *i*-th component state reduction
CDRI	CDRI is similar to SI and take into account the probability of the *i*-th component state reduction
DIRI	DIRI indicates the probability of a system performance level change depending on the change of any component state

The reduction of performance level and component state for BSS is defined as the system failure. Therefore the Eq. (5) for BSS is represented as:

$$\partial\phi(1 \to 0)/\partial x_i(1 \to 0) = \begin{cases} 1, \text{ if } \phi(1_i, \boldsymbol{x}) = 1 \text{ and } \phi(0_i, \boldsymbol{x}) = 0 \\ 0, \quad \text{other} \end{cases}. \tag{6}$$

Derivatives (4)–(6) allow calculation boundary states of system reliability depending on the *i*-th component state change that are agree with vector state:

$$\boldsymbol{x} = (x_1 \ldots x_{i-1}, a_i \to a_i - 1, x_{i+1} \ldots x_n). \tag{7}$$

The boundary states are one of basic conception in the reliability analysis and used in different mathematical methods for the computation of reliability indices and measures. For example boundary state is principal item in the investigation based on Fault Tree [6, 11, 22]. These states are considered and analysed in the method of *Failure Models and Effect Analysis* (FMEA) [11, 27]. The boundary states are used in Importance analysis for the computation of the *Importance Measures* (IM) [32, 36]. Importance analysis allows examining different aspects of reliability changes and the uncertainty in the system. IM quantifies the criticality of a particular component within the system. They have been widely used as tools for identifying system weaknesses, and to prioritise reliability improvement activities.

The most used IMs as *Structural Importance* (SI), *Birnbaum importance* (BI), *Critical importance* (CI), *Component Dynamic Reliability Indices* (CDRI) and *Dynamic Integrated Reliability Indices* (DIRI) are shown in Table 2 [17, 36].

The Direct Partial Logic Derivative is one of possible approaches for calculation of IMs [36]. In this paper we develop unify method for calculation of the IMs (Table 2) based on the Direct Partial Logic Derivative (5). These measures can be used for the estimation of healthcare system component that has maximal influence to system functioning (system availability).

Therefore last step in the estimation of healthcare system according to Fig. 1 is the measuring of the system behaviour that is calculation of IMs.

2.4 Measures of Healthcare System

Reliability function for Binary-State System is one of basic measures of reliability and this measure is defined as probability of system function without failure during given period of time. In paper [17] this measure $R(t)$ for MSS is interpreted as is the probability of the system being operational throughout the interval [0, t):

$$R(t) = \Pr\{T \geq t, \phi(\boldsymbol{x}) > 0\}. \tag{8}$$

In stationary state instead of reliability function the measure as a system availability is used. A system availability for BSS is probability of a system functioning. But for MSS there are some levels of system performance and reliability analysis of this system needs to include estimation of probability of system to be in every of these performance state. Therefore there are some definitions of system availability for MSS. One of them allows to presented probability of MSS to be in state, that isn't less than performance level j ($0 \leq j \leq M - 1$) [21]:

$$A(j) = \Pr\{\phi(\boldsymbol{x}) \geq j\}. \tag{9}$$

There is one more interpretation of MSS availability in [36] that in paper [17] is named as probability of MSS state. This measure is defined as probability of system reliability that is equal to the performance level j:

$$A_j = \Pr\{\phi(\boldsymbol{x}) = j\}. \tag{10}$$

The correlation of measures (9) and (10) is defined as:

$$A(j) = \sum_{r=1}^{j} A_r. \tag{11}$$

According to (11) the measure (10) is more exact and allows computing other measures as MSS availability (9) and MSS unreliability:

$$F = A_0 = 1 - \sum_{j=1}^{M-1} A_j. \tag{12}$$

Therefore we use in the development of methods for MSS reliability analysis the conception of system availability defined by (10).

The system availability A_j of the MSS is calculated based on probabilities of components states (3):

$$A_j = \sum_{\phi(\boldsymbol{x})=j} p_{1,s_1} \cdot p_{2,s_2} \cdot \ldots \cdot p_{n,s_n}, \tag{13}$$

where $(s_1, \quad s_2 \ldots s_n)$ are values of vector state $\boldsymbol{x} = (x_1, \quad x_2 \ldots x_n)$ for which $\phi(x_1$ $x_2 \ldots x_n) = j$.

The measures (9), (10) and (12) characterise system availability in general. Therefore they can be used for the availability estimation of healthcare system in stationary state. And these measures don't take into account of the influence of components states changes to a system performance level. There is other group of reliability measures that permit to investigate these aspects of system behaviour. These measures are IMs (Table 2) that are provided under importance analysis.

3 Importance Analyses

The importance analysis is part of reliability engineering that allows investigating the structural and topological aspects of the system in point of view of the reliability/availability. The IMs permit to indicate a system component with maximal/minimal influence to system reliability/availability. In particular, these measures indicate the probability of system performance level change caused by the change of fixed component state. Consider some of IMs (Table 2) in more details below.

In this paper investigated system is specified and it is a healthcare system. As a rule a healthcare system has specific properties that permit to consider this system as a coherent system. A coherent system has assumptions [2]:

(a) All system components are relevant to the system;
(b) The system structure function is monotone non-decreasing: $\phi(x_1, \ldots, 1, \ldots, x_n) \quad \neq \phi(x_1, \ldots, 0, \ldots, x_n)$;
(c) The component state decreases to one only: from s to $s - 1$;

These assumptions will be used in the definition and computation of IMs.

In addition need to note, that analysis of a coherent system performance level decrease and increase are similar. Therefore the analysis of a system performance level decrease will be considered in this paper only.

3.1 Structural Importance

SI is one of the simplest measures that focuses on the topological and structural properties and aspects of a system. According to the definition of this measure for BSS in paper [1], this measure determines the proportion of working states of the system in which the working of the i-th component makes the difference between system failure and its working. The generalization of the conception of SI for MSS has to take into account all possible changes of system performance levels and for MSS is defined as proportion of system state in which the change of the i-th component state makes difference in system performance level from j to h and its performance level j. Because we consider a coherent system, the component state changes from s to $s-1$ according to assumption (c) for a coherent system, the mathematical definition of SI is:

$$IS_i^{s,j\to h} = \frac{\rho_i^{(s,j\to h)}}{\rho_{s,j}}, \tag{14}$$

where $\rho_i^{(s,j\to h)}$ is a number of system states when the change component state from s to $s-1$ results in the system reliability change from j to h; $\rho_{s,j}$ is number of the states for which $\phi(s_i, x) = j$ and is calculated based on the structure function.

The number $\rho_i^{(s,j\to h)}$ can be computed as the number of nonzero values of the Direct Partial Logic Derivative (5).

3.2 Birnbaum Importance

SI (14) investigates influence of system component to system performance level based on a system structure or topology only. But this measure doesn't take into account the probability of state of system components. This disadvantage can be eliminated by other importance measure as BI. BI is one of basic IMs and this measure is defined as the probability that a system is sensitive to inoperative state of the i-th system component in case of BSS [2]. In paper [38] new equation for the BI calculation has been proposed based on Direct Partial Boolean Derivatives:

$$BI_i = \Pr\{\partial\phi(1 \to 0)/\partial x_i(1 \to 0) = 1\}. \tag{15}$$

The Eq. (15) for calculation of BI can be generalized for MSS analysis:

$$IB_i^{s,j\to h} = \Pr\{\partial\phi(j \to h)/\partial x_i(s \to s-1) \neq 0\}. \tag{16}$$

The definition of BI for MSS (16) is indicated as the probability that a system performance level j is sensitive to change from s to $s-1$ of state of the i-th component. In practical, the Eq. (16) is calculated as the probabilities of non-zero values of Direct Partial Logic Derivative $\partial\phi(j\to h)/\partial x_i(s\to s\text{–}1)$.

3.3 Critical Importance

BI (16) describes the influence of a i-th component state change from s to $s-1$ on a system's performance level j, but doesn't take into account the probability of this component's state. CI adjusts it and is defined as the quantitative measure that the i-th system component is relevant to the system's performance level j if it has changed state from s to $s-1$ [36]:

$$CI_i^{s,j \to h} = BI_i^{s,j \to h} \cdot \frac{p_{i,s-1}}{A_j}, \tag{17}$$

where $IB_i^{s,j \to h}$ is the i-th system component BI measure (16); $p_{i,s-1}$ is probability of the i-th component state $s-1$ (3) and A_j is the probability of system's performance level j (10).

3.4 Dynamic Reliability Indices

Dynamic Reliability Indices (DRIs) have been introduced in paper [35]. DRIs allow the estimation of a component relevant to system failure. There are two groups of DRI: *Component Dynamic Reliability Indices* (CDRI) and *Dynamic Integrated Reliability Indices* (DIRI).

CDRI indicates the influence of the i-th component's state change from s to $s-1$ on a system's performance level j and is similar to the definition of SI, but CDRI includes two probabilities: (i) the probability of a system's performance level change caused by the i-th component's state change and (ii) the probability of the component state:

$$CDRI_i^{s,j \to h} = SI_i^{s,j \to h} \cdot p_{i,s-1}, \tag{18}$$

where SI_i is defined by (14); $p_{i,s-1}$ is probability of the i-th component state $s-1$ (3).

DIRI has similar conception but this index indicates the influence of any component state change to the system performance level j. Therefore DIRI is defined as the probability of the performance level change from j to h that is caused by one of the system components state change (one of n):

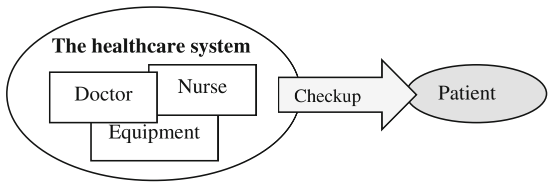
$$DIRI^{s,j \to h} = \sum_{i=1}^{n} CDRI_i^{s,j \to h} \prod_{\substack{q=1 \\ q \neq i}}^{n} (1 - CDRI_q^{s,j \to h}). \tag{19}$$

4 Importance Analyses of a Healthcare System

Consider the proposed methodology of a healthcare system analysis based on hand-calculation example. The investigated system is shown in Fig. 4. This system includes three components: doctor, nursing and equipment for patient checkup. The principal problem of this system is medical error that is interpreted as the system fault. There are 6 possible problems caused the medical error of this system that are most common type of problems according to [8, 9, 12]. The typical methods of reliability analysis are able to investigate the human error and technical problem in such system separately. The proposed method can be used to estimate this system as a whole.

According to proposed method the first step in the system reliability analysis is definition of number of system performance levels (Sect. 2.1). The system has fife components ($n = 5$) that are conformed to typical problem of patient's checkup (Table in Fig. 4). We will use three performance level for the investigate system (M = 3) and two state for the system components states ($m_1 = m_2 = m_3 = m_4 = m_5 = 2$). The values and contents of these variables are presented in Table 3. The state of this healthcare system is interpreted as: the fault (value 0) in case of a patient given wrong medication or incorrect amount (fatal error of treatment); the partial work (value 1)

Fig. 4 The example of the healthcare system and possible medical error



Problems of checkup	Doctor	x_1	Misdiagnosis	**Medical error**
		x_2	Haste	
	Nurse	x_3	Incorrect interpretation of instructions	
		x_4	Inappropriate abbreviation	
	Equipment	x_5	Equipment problem	

Table 3 The healthcare system components

The system components	Values of components	
	0	1
x_1	Misdiagnosis	Correct diagnosis
x_2	Haste	Sufficient time
x_3	Incorrect interpretation of doctor instructions	Incorrect interpretation of doctor instructions
x_4	Inappropriate abbreviation	Appropriate abbreviation
x_5	Equipment fault	Working of equipment

Table 4 The structure function of the MSS for the healthcare system in Fig. 4

x_4x_5	$x_1x_2x_3$							
	0 0 0	0 0 1	0 1 0	0 1 1	1 0 0	1 0 1	1 1 0	1 1 1
0 0	0	0	0	0	1	1	1	2
01	0	0	0	0	1	1	1	2
10	0	0	0	0	1	1	2	2
11	0	0	1	1	1	2	2	2

if incorrect medical work doesn't cause a patient health; the perfect work (value 2) agrees with the correct treatment of a patient.

The second step of the method for the system reliability estimation is the definition of the mathematical model of the system and the mathematical method for the calculation of reliability indices. In the Sect. 2.2 the analysis of the possible mathematical methods and models have been considered. And the structure based methods for the analysis of system reliability have been proposed. Therefore the investigated healthcare system is represented by the structure function. This structure function is in Table 4.

Consider the importance analysis of this healthcare system. The reliability indices based on the healthcare system representation in the form of the structure function are calculated according to (10), (12) and (14)–(17).

This system availability (10) and system unavailability (12) for this system is calculated based on the structure function (Table 4) and the probabilities of the system components states (Table 5):

$$F = A_0 = p_{0,1}p_{0,2} + p_{0,1}p_{1,2}(p_{0,4} + p_{1,4}p_{0,5}) = 0.01472;$$

$$A_1 = p_{0,1}p_{1,2}p_{1,4}p_{1,5} + p_{1,1}p_{0,2}p_{0,3} + p_{1,1}p_{0,2}p_{1,3}(p_{0,4} + p_{1,4}p_{0,5}) \\ + p_{1,1}p_{1,2}p_{0,3}p_{0,4} = 0.09424;$$

$$A_2 = p_{1,1}p_{0,2}p_{1,3}p_{1,4}p_{1,5} + p_{1,1}p_{1,2}(p_{0,3} \quad p_{1,4} + p_{1,3}) = 0.89104.$$

Table 5 The healthcare system components probabilities

Component state	x_1	x_2	x_3	x_4	x_5
0	0.05	0.20	0.15	0.10	0.02
1	0.95	0.80	0.85	0.90	0.98

Continue the analysis of the healthcare system and calculate IMs (14)–(17). The first of these indices is SI (14). Values of SI calculated based on the Direct Partial Logical Derivatives (5). These derivatives for the healthcare system are presented in Table 5. Iintermediate number of $\rho_i^{(s,j\to h)}$ and $\rho_{s,j}$ in (15) are calculate based on these derivatives (Table 6) and the structure function (Table 4) accordantly. Values of these numbers and SI measures for all system components are shown in Table 7.

SI (Table 7) permits to estimate the influence of doctor decision, nurse work and equipment function to a patient treatment. The fatal influence is evaluated by SIs $IS_i^{(1,1\to0)}$ and $IS_i^{(1,2\to0)}$ that analysis of the healthcare system failure. The measure $IS_1^{(1,1\to0)}$ has maximal value, therefore the first component has important influence to the system functioning. It means that doctor's error (misdiagnosis) has maximal influence to correct treatment of patient. And nurse incorrect interpretation of doctor's instruction has not fatal influence to a patient treatment for this healthcare system structure (organization), because $IS_3^{(1,1\to0)} = 0$ and $IS_3^{(1,2\to0)} = 0$.

Therefore SI allows investigate influence of fixed component state change to change of system performance level. But this measure doesn't take into account the probabilities of component state. This disadvantage is absent in importance analysis by BI (16).

For example, let us continue the analysis of the healthcare system in Fig. 4 and calculate BI for this system. This measure according to (15) is computed based on Direct Partial Logic Derivatives. These derivatives for the healthcare system are in Table 6. Consider the influence of misdiagnosis to fatal error in treatment of patient that is defined by BI for the first component $IB_1^{(1,1\to0)}$ and $IB_1^{(1,2\to0)}$:

$$\begin{aligned} IB_1^{(1,1\to0)} &= \Pr\{\partial\phi(1\to0)/\partial x_1(1\to0)=1\} \\ &= p_{0,2}p_{0,3}+p_{0,2}p_{1,3}p_{0,4}+p_{0,2}p_{1,3}p_{1,4}p_{0,5}+p_{1,2}p_{0,3}p_{0,4}=0.0621 \end{aligned}$$

$$\begin{aligned} IB_1^{(1,2\to0)} &= \Pr\{\partial\phi(2\to0)/\partial x_1(1\to0)=1\} = p_{0,2}p_{1,3}p_{1,4}p_{1,5}+p_{1,2}p_{0,3}p_{1,4}p_{0,5} \\ &= 0.1521 \end{aligned}$$

BIs for all system components are in Table 8. These measures are calculated similar to $IB_1^{(1,1\to0)}$ and $IB_1^{(1,2\to0)}$ based on (16).

The analysis of data in Table 8 allows indicate the most possible problem (maximum value of BI) of the healthcare system is incorrect treatment without problem of patient health caused by misdiagnosis ($IB_1^{(1,2\to1)} = 0.7056$). A misdiagnosis isn't importance for fatal error of patient treatment ($IB_1^{(1,1\to0)}$ and $IB_1^{(1,2\to0)}$).

CI (17) is similar to BI but takes into account the probability of the i-th component state change from s to $s-1$. These measures for the healthcare system (Fig. 4) are shown in Table 9. Analysis of these measures allows defining that the doctor's haste is more possible reason of menace of a patient health ($IC_2^{(1,1\to0)} = 0.5992$). This reason is important for non-fatal problem of a patient treatment too, because the value of CI measure is maximal for the second component in case of the system performance level change from level 2 to 1 ($IC_2^{(1,2\to1)} = 0.4744$).

Table 6 Direct partial logical derivatives $\partial\phi(j \rightarrow j\text{–}1)/\partial x_1(1\rightarrow 0)$ for the healthcare system in Fig. 4

x_2	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
x_3	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
x_4	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
x_5	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
$\partial\phi(1\rightarrow 0)/\partial x_1(1\rightarrow 0)$	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**0**	**1**	**1**	**0**	**0**	**0**	**0**	**0**	**0**
$\partial\phi(2\rightarrow 1)/\partial x_1(1\rightarrow 0)$	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**1**	**0**	**0**	**0**	**1**
$\partial\phi(2\rightarrow 0)/\partial x_1(1\rightarrow 0)$	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**1**	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**
x_1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
x_3	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
x_4	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
x_5	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
$\partial\phi(1\rightarrow 0)/\partial x_2(1\rightarrow 0)$	**0**	**0**	**0**	**1**	**0**	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
$\partial\phi(2\rightarrow 1)/\partial x_2(1\rightarrow 0)$	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**1**	**1**	**1**	**1**	**1**	**0**
$\partial\phi(2\rightarrow 0)/\partial x_2(1\rightarrow 0)$	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
x_1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
x_2	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
x_4	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
x_5	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
$\partial\phi(1\rightarrow 0)/\partial x_3(1\rightarrow 0)$	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
$\partial\phi(2\rightarrow 1)/\partial x_3(1\rightarrow 0)$	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**1**	**1**	**1**	**0**	**0**
$\partial\phi(2\rightarrow 0)/\partial x_3(1\rightarrow 0)$	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**

(continued)

Table 6 (continued)

x_1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
x_2	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
x_3	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
x_5	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
$\partial\phi(1\to0)/\partial x_4(1\to0)$	**0**	**0**	**0**	**0**	**0**	**1**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
$\partial\phi(2\to1)/\partial x_4(1\to0)$	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**1**	**1**	**1**	**0**	**0**
$\partial\phi(2\to0)/\partial x_4(1\to0)$	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
x_1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
x_2	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
x_3	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
x_4	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
$\partial\phi(1\to0)/\partial x_5(1\to0)$	**0**	**0**	**0**	**0**	**0**	**1**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
$\partial\phi(2\to1)/\partial x_5(1\to0)$	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**1**	**0**	**0**	**0**	**0**
$\partial\phi(2\to0)/\partial x_5(1\to0)$	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**

Table 7 Structural importance for the healthcare system

i	$\rho_i^{(s,j\to h)}$			$\rho_{s,j}$			$IS_i^{(s,j\to h)}$		
	$\rho_i^{(1,1\to 0)}$	$\rho_i^{(1,2\to 1)}$	$\rho_i^{(1,2\to 0)}$	$\rho_{1,1}$	$\rho_{1,2}$	$\rho_{1,2}$	$IS_i^{(1,1\to 0)}$	$IS_i^{(1,2\to 1)}$	$IS_i^{(1,2\to 0)}$
1	9	2	2	9	7	7	1.0000	0.2857	0.2857
2	2	5	0	4	6	6	0.5000	0.8333	0
3	0	3	0	4	5	5	0	0.6000	0
4	2	3	0	5	5	5	0.4000	0.6000	0
5	2	1	0	6	4	4	0.3300	0.2500	0

Table 8 BI for the healthcare system Fig. 4

i	$IB_i^{(1,1\to 0)}$	$IB_i^{(1,2\to 1)}$	$IB_i^{(1,2\to 0)}$
1	0.0621	0.7056	0.1521
2	0.0441	0.2235	0
3	0	0.2436	0
4	0.0392	0.2723	0
5	0.0360	0.1453	0

Table 9 CI for the healthcare system Fig. 4

i	$IC_i^{(1,1\to 0)}$	$IC_i^{(1,2\to 1)}$	$IC_i^{(1,2\to 0)}$
1	0.2108	0.3744	0.0085
2	0.5992	0.4774	0
3	0	0.3877	0
4	0.2663	0.0489	0
5	0.2890	0.0308	0

Consider next indices for the healthcare system. It is CDRI (18) that indicates the influence of the i-th component's state change from s to $s-1$ on a system's performance level j. CDRIs for the healthcare system are presented in Table 10. The comparison of SI (Table 7) and CDRI (Table 10) illustrate the influence of component state probability to the importance of the i-th component. So the influence of doctor's error (misdiagnosis) is less in case if the probability of this error is took into account: $CDRI_1^{(1,1\to 0)} = 0.0500$ but $IS_1^{(1,1\to 0)} = 1$. CDRIs show that a doctor's haste is more possible for fatal error in treatment that misdiagnosis: $CDRI_2^{(1,1\to 0)} = 0.1000$. This problem is important for non-fatal error of patient treatment too, because $CDRI_i^{(1,2\to 1)}$ has maximal value for second component (doctor's haste): $CDRI_2^{(1,2\to 1)} = 0.1667$

DIRI estimates the influence of any component state change to the system performance level j: $DIRI^{1,1\to 0} = 0.1729$, $DIRI^{1,2\to 1} = 0.2673$ and $DIRI^{1,2\to 0} = 0.0143$. These indices indicate that incorrect medical work (non-fatal problem for a

Table 10 CDRI for the healthcare system in Fig. 4

i	$CDRI_i^{(1,1\to 0)}$	$CDRI_i^{(1,2\to 1)}$	$CDRI_i^{(1,2\to 0)}$
1	0.0500	0.0143	0.0143
2	0.1000	0.1667	0
3	0	0.0900	0
4	0.0400	0.0600	0
5	0.0066	0.0050	0

patient treatment) is more possible in case of any problem of the healthcare system components, because DIRI has maximal value for system performance level change from level two to one ($DIRI^{1,2\to 1} = 0.2673$).

Analysis of all IMs (Tables 7, 8, 9 and 10) shows that two components of the investigated healthcare system (Fig. 4) has maximal influence to a patient correct treatment and heal that are doctor's errors (misdiagnosis and haste). The misdiagnosis is more important in case of fatal problem in a patient treatment (Tables 8 and 9). But the doctor's hast is more possible for this system because measures CI (Table 9) and CDRI (Table 10) have maximal values for the second component and these measures take into account the probability of component state too. DIRIs allow indicating the incorrect medical work (non-fatal problem for a patient treatment) as more possible problem of this system. Therefore all IMs allow estimate different aspect of system performance level change depending on changes of components states.

5 Conclusions

Reliability analysis of a healthcare system is important problem in medicine. The application of information technologies in medicine supposes high level of reliability of medical equipment and devices. But only reliable technics can't ensure a correct patient treatment. The human factor must be included in reliability analysis too. Therefore a healthcare system is interpreted as the combination of technical and human components to assure a correct treatment of a patient [40].This definition of a healthcare system needs the development new conception and methods of reliability analysis. These methods must to permit to investigate a healthcare system reliability based on the united background without separation of system to independent parts. The theoretical aspects of this conception have been discussed in [37, 39]. In this paper some practical positions of this conception are considered, in particular the mathematical interpretation of investigated system and some of possible techniques for reliability analysis of a healthcare system (as importance analysis).

The first and principal step in a healthcare system reliability analysis is the development of the mathematical representation of this system (development of mathematical model). The design of a healthcare system model has two specifics. The first of them it is definition of number of a system performance levels. There are two possibilities for interpretation of investigation system depending of number of system performance level: BSS or MSS. BSS allows investigation only two performance level as working and failure that isn't sufficient for detail estimation of a healthcare system. Therefore MSS is more useful mathematical model that permits to analyse some changes in healthcare system reliability behaviour. The second specific in the modelling of investigated system is defined by mathematical methods that are used for calculation of reliability indices and measures. In this paper the structure function is considered because this mathematical interpretation of investigated system allows defining the mathematical model for system with high complexity. As result, the mathematical representation of investigated healthcare system is proposed in form of MSS structure function according to (1).

The second step in reliability analysis of healthcare system is calculation of indices and measures for quantitative reliability analysis. In this paper IMs (Table 2) are proposed for analysis and estimation of a healthcare system and algorithms for the calculation these measures are presented in Sect. 3. The simple hand calculation example of a healthcare system (Sect. 4) illustrates the efficiency of these measures application for reliability analysis.

Therefore in this paper we propose and consider the mathematical representation of a healthcare system in form of MSS structure function and analysis of this system based on IMs. These measures indicate healthcare components that are more important and principal for system correct functioning. And priority control of these components allows ensuring high level of a healthcare system and correct treatment of a patient.

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